Surface Areas and Volumes

Exercise 6.1

I. Very Short Answer Type Questions [1 mark]

1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

(1) The radius of a sphere (in cm) whose volume is 12π cm^3, is [CBSE Standard 2020]

(a) 3 (b) $3\sqrt{3}$ (c) $3^{2/3}$ (d) $3^{1/3}$

(2) If the surface area of a sphere is 144π, then its radius is

(a) 6 cm (b) 8 cm (c) 12 cm (d) 10 cm

(3) The edge of a cube whose volume is equal to that of a cuboid of dimensions 8 cm × 4 cm × 2 cm is

(a) 6 cm (b) 4 cm (c) 2 cm (d) 8 cm

(4) If the radii of two spheres are in the ratio 2 : 3, then the ratio of their respective volumes is

(a) 8 : 27 (b) 3 : 5 (c) 7 : 24 (d) 5 : 14

(5) The edge of a cube whose volume is $8x^3$ is

(a) x (b) $2x$ (c) $4x$ (d) $8x$

(6) If the volume of a 7 cm high right circular cylinder is 448π cm^3, then its radius is equal to

(a) 2 cm (b) 4 cm (c) 6 cm (d) 8 cm

2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

(1) **Assertion (A):** Total surface area of the cylinder having radius of the base 14 cm and height 30 cm is 3872 cm^2.
Reason (R): If r be the radius and h be the height of the cylinder, then total surface area = $(2\pi rh + 2\pi r^2)$.

(2) **Assertion (A):** If the height of a cone is 24 cm and diameter of the base is 14 cm, then the slant height of the cone is 15 cm.
Reason (R): If r is the radius and h the height of the cone, then slant height = $\sqrt{h^2 + r^2}$.

3. Answer the following.

(1) Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere? [Delhi 2017]

(2) Total surface area of a cube is 216 cm^2. Find its volume. [CBSE SP 2012]

(3) If a solid right-circular cone of height 24 cm and base radius 6 cm is melted and recast in the shape of a sphere, find the radius of the sphere. [CBSE SP 2012]

(4) Find the curved surface area of a right-circular cone of height 15 cm and base diameter 16 cm. [CBSE 2011]

(5) Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes? [CBSE Standard 2020]

(6) Find the radius of the sphere whose surface area is 36 π cm^2.

(7) 12 solid spheres of the same radii are made by melting a solid metallic cylinder of base diameter 2 cm and height 16 cm. Find the diameter of the each sphere. [CBSE Standard SP 2020-21]

II. Short Answer Type Questions - I [2 Marks]

4. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder. [NCERT]

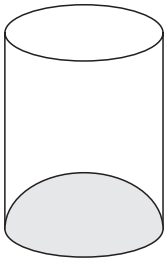
5. Three solid metal cubes of edges 6 cm, 8 cm and 10 cm are melted and recasted into a single solid cube. Find the length of the edge of the cube so obtained. **[Imp.]**

6. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire. [NCERT] **[Imp.]**

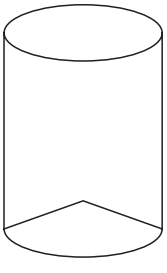
7. Two spheres of same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere. [CBSE 2015]

8. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform. [NCERT]

9. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice.
Aisha, a juice seller, was serving juice to her customers in two types of glasses. Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.



First type: A glass with hemispherical raised bottom.



Second type: A glass with conical raised bottom of height 1.5 cm.

Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much? [CBSE Standard SP 2019-20]

10. A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes. [CBSE Standard 2020]

11. The volume of a right circular cylinder with its height equal to the radius is $25\frac{1}{7}$ cm^3. Find the height of the cylinder. [Use $\pi = \frac{22}{7}$] [CBSE Standard 2020]

III. Short Answer Type Questions - II [3 Marks]

12. A solid sphere is melted and recasted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 4 cm, its height 24 cm and thickness 2 cm; find the radius of the sphere. **[Imp.]**

13. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? [CBSE 2018]

14. A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere [NCERT] **[Imp.]**

15. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm? [CBSE 2011]

16. The $\frac{3}{4}$th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel. [Delhi 2017]

17. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm standing water is needed? [NCERT] [Delhi 2019, SP 2018]

18. In a hospital, used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital whose length is 25 m and breadth is 20 m. If tank is filled completely then what will be the height of standing water used for irrigating the park? [Delhi 2017]

19. In a rain-water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and height 3.5 m. If the tank is full, find the rainfall in cm.

20. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a solid sphere. Find the radius of the resulting sphere.

IV. Long Answer Type Questions [5 Marks]

21. A well, whose diameter is 3 m, has been dug 21 m deep and the earth dug out is used to form an embankment 4 m wide around it. Find the height of the embankment. **[Imp.]**

22. A farmer connects a pipe of internal diameter 20 cm from the canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled? [NCERT]

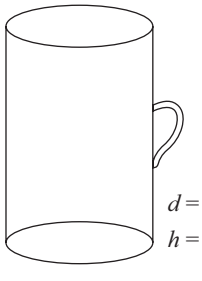
23. Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from an underground tank which is in the shape of a cuboid. The underground tank has dimensions 1.57 m × 1.44 m × 95 cm. The overhead tank has its radius of 60 cm and its height is 95 cm. Find the height of the water left in the underground tank after the overhead tank has been completely filled with water from the underground tank which had been full. Compare the capacity of the overhead tank with that of the underground tank. [Use $\pi = 3.14$] [NCERT]

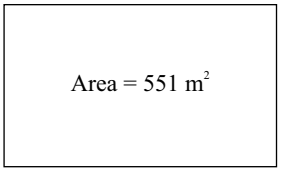
24. In a cylindrical vessel of radius 10 cm, containing some water, 9000 small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius 0.5 cm, then find the rise in the level of water in the vessel. [CBSE Standard 2020]

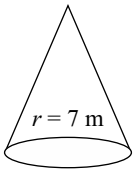
25. Water is flowing through a cylindrical pipe of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm at the rate of 0.7 m/sec. By how much will the water rise in the tank in half an hour? [CBSE Standard SP 2020-21]

Case Study Based Questions

I. Adventure camps are the perfect place for the children to practise decision making for themselves without parents and teachers guiding them every move. Some students of a school reached for adventure at Sakleshpur. At the camp, the waiters served some students with a welcome drink in a cylindrical glass while some students in a hemispherical cup whose dimensions are shown below. After that they went for a jungle trek. The jungle trek was enjoyable but tiring. As dusk fell, it was time to take shelter. Each group of four students was given a canvas of area 551 m^2. Each group had to make a conical tent to accommodate all the four students. Assuming that all the stitching and wasting incurred while cutting, would amount to 1 m^2, the students put the tents. The radius of the tent is 7 m.







1. The volume of cylindrical cup is
(*a*) 295.75 cm^3 (*b*) 7415.5 cm^3 (*c*) 384.88 cm^3 (*d*) 404.25 cm^3

2. The volume of hemispherical cup is
(*a*) 179.67 cm^3 (*b*) 89.83 cm^3 (*c*) 172.25 cm^3 (*d*) 210.60 cm^3

3. Which container had more juice and by how much?
(*a*) Hemispherical cup, 195 cm^3 (*b*) Cylindrical glass, 207 cm^3
(*c*) Hemispherical cup, 280.85 cm^3 (*d*) Cylindrical glass, 314.42 cm^3

4. The height of the conical tent prepared to accommodate four students is
(*a*) 18 m (*b*) 10 m (*c*) 24 m (*d*) 14 m

5. How much space on the ground is occupied by each student in the conical tent
(*a*) 54 m^2 (*b*) 38.5 m^2 (*c*) 86 m^2 (*d*) 24 m^2

Answers and Hints

1. (1) (*c*) $3\frac{2}{3}$ (1) (2) (*a*) 6 cm (1)
(3) (*b*) 4 cm (1) (4) (*a*) 8 : 27 (1)
(5) (*b*) 2*x* (1) (6) (*d*) 8 cm (1)

2. (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Total surface area $= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$

$= 2 \times \frac{22}{7} \times 14(30 + 14)$

$= 88(44) = 3872\ cm^2$

(2) (*d*) Assertion (A) is false but reason (R) is true.

Slant height $= \sqrt{\left(\frac{14}{2}\right)^2 + (24)^2}$

$= \sqrt{49 + 576}$

$= \sqrt{625} = 25$

3. (1) $\frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2}$ units

$\therefore$ $d = 9$ units (1)

(2) $6l^2 = 216 \Rightarrow l^2 = 36$

$\Rightarrow$ $l = 6$

$\therefore$ Volume of cube $= l^3 = (6)^3 = 216\ cm^3$ (1)

(3) Volume of cone = volume of sphere

$\therefore \frac{1}{3}\pi 6^2 \times 24 = \frac{4}{3}\pi r^3$

$\Rightarrow 864 = 4r^3$

$\Rightarrow r^3 = 216$

$\Rightarrow r = 6$ cm

So, radius of sphere = 6 cm (1)

(4) Slant height of cone, $l = \sqrt{8^2 + 15^2}$

($\because$ Diameter = 16 cm)

$\Rightarrow l = 17$ cm

$\therefore$ CSA of cone $= \pi rl = \pi \times 8 \times 17$

$= 136\ \pi\ cm^2$ (1)

(5) Let h_1 and h_2 be the heights of two cones.

Then, $\frac{h_1}{h_2} = \frac{1}{3}$

Also, let r_1 and r_2 be the radii of two cones.

Then, $\frac{r_1}{r_2} = \frac{3}{1}$

Now,

$$\frac{V_1 \text{ (Volume of first cone)}}{V_2 \text{ (Volume of second cone)}} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{r_1^2}{r_2^2} \times \frac{h_1}{h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{3}{1}\right)^2 \times \left(\frac{1}{3}\right) = \frac{9}{1} \times \frac{1}{3}$$

$$= \frac{3}{1} = 3 : 1$$

(6) Let *r* be the radius of sphere.

Then, $4\pi r^2 = 36\pi$

$\Rightarrow r^2 = \frac{36\pi}{4\pi} = 9$

$\Rightarrow r = 3$ cm

(7) $\pi R^2 H = 12 \times \frac{4}{3}\pi r^3$

$$1 \times 1 \times 16 = \frac{4}{3} \times r^3 \times 12 \qquad (\frac{1}{2})$$
$$r^3 = 1^3$$
$$r = 1$$
$$d = 2 \text{ cm} \qquad (\frac{1}{2})$$

4. Radius of the sphere $(r_1) = 4.2$ cm

$\therefore$ Volume of the sphere

$$= \left(\frac{4}{3}\pi r_1^3\right) = \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \text{ cm}^3$$

Radius of the cylinder $(r_2) = 6$ cm

Let 'h' be the height of the cylinder

$\therefore$ Volume of the cylinder $= \pi r_2^2 h$

$$= \frac{22}{7} \times 6 \times 6 \times h \text{ cm}^3 \qquad (1)$$

Since, Volume of the metallic sphere

= Volume of the cylinder.

$$\Rightarrow \quad \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10}$$
$$= \frac{22}{7} \times 6 \times 6 \times h$$
$$\Rightarrow \quad h = \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \times \frac{7}{22} \times \frac{1}{6} \times \frac{1}{6} \text{ cm}$$
$$= \frac{4 \times 7 \times 7 \times 14}{10 \times 10 \times 10} = \frac{2744}{1000}$$
$$= 2.744 \text{ cm} \qquad (1)$$

5. Volume of single cube so obtained

$$= (6^3 + 8^3 + 10^3) \text{ cm}^3$$
$$\Rightarrow \quad l^3 = 216 + 512 + 1000 = 1728 \Rightarrow l = 12 \text{ cm} \qquad (2)$$

6. Volume of rod = volume of wire

$$\therefore \pi \times \left(\frac{1}{2}\right)^2 \times 8 = \pi \times r^2 \times 1800$$
$$\Rightarrow 2\pi = \pi r^2 \times 1800 \quad \Rightarrow \quad r^2 = \frac{1}{900}$$
$$\Rightarrow \quad r = \frac{1}{30} \text{ cm} \qquad (1)$$

So, thickness of wire

= diameter of cross-section of wire

$$= \frac{1}{30} \times 2 = \frac{1}{15} \text{ cm} = 0.67 \text{ mm (approx)}$$
(1)

7. 12 cm (2)

8. Volume of well $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20$

$$= 22 \times 7 \times 5 \text{ m}^3$$

$\Rightarrow$ Volume of the earth taken out $= 22 \times 7 \times 5 \text{ m}^3$ (1)

Now this earth is spread out to form a cuboial platform having length = 22 m, breadth = 14 m

Let 'h' be the height of the platform.

$\therefore$ Volume of the platform $= 22 \times 14 \times h \text{ m}^3$

$\therefore \quad 22 \times 14 \times h = 22 \times 7 \times 5$

$$\Rightarrow \quad h = \frac{22 \times 7 \times 5}{22 \times 14} = \frac{5}{2} = 2.5 \text{ m}$$

Thus, the required height of the platform is 2.5 m. (1)

9. Capacity of first glass

$$= \pi r^2 H - \frac{2}{3}\pi r^3$$
$$= \pi \times 9(10 - 2) = 72\pi \text{ cm}^3 \qquad (1)$$

Capacity of second glass

$$= \pi r^2 H - \frac{1}{3}\pi r^2 h = \pi \times 3 \times 3(10 - 0.5)$$
$$= 85.5\pi \text{ cm}^3$$

$\therefore$ Suresh got more quantity of juice. (1)

10. Let the radii of a cone and a cylider be r

And, let the height of the cylinder be h

Then, the height of the cone = $3h$

Now, volume of cone $(V_1) = \frac{1}{3}\pi r^2 (3h) = \pi r^2 h$

and volume of cylinder $(V_2) = \pi r^2 h$ (1)

$$\therefore \quad \frac{V_1}{V_2} = \frac{\pi r^2 h}{\pi r^2 h}$$
$$\Rightarrow \quad V_1 : V_2 = 1 : 1 \qquad (1)$$

11. Let the height and radius of cylinder be r

[$\because$ Height of cylinder = radius of cylinder]

Then, Volume of cylinder $= \pi r^2 h$

$$\Rightarrow \quad \pi r^2 \times r = 25\frac{1}{7} \quad \Rightarrow \quad \pi r^3 = \frac{176}{7} \qquad (1)$$
$$\Rightarrow \quad \frac{22}{7} \times r^3 = \frac{176}{7} \quad \Rightarrow \quad r^3 = \frac{176}{22}$$
$$\Rightarrow \quad r = 2 \text{ cm} \qquad (1)$$

12. Volume of sphere = volume of hollow cylinder

$$\Rightarrow \quad \frac{4}{3}\pi \times r^3 = \pi \times 24 \times (4^2 - 2^2) \qquad (1)$$
$$\Rightarrow \quad \frac{4\pi r^3}{3} = 288\pi \quad \Rightarrow \quad r^3 = \frac{288\pi \times 3}{4\pi} \qquad (1)$$
$$\Rightarrow \quad r^3 = 216 \quad \Rightarrow \quad r = 6 \text{ cm} \qquad (1)$$

13. Radius of conical heap = 12 m

$$\text{Volume of rice} = \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 \text{ m}^2$$
$$= 528 \text{ m}^3 \qquad (1)$$

Area of canvas cloth required $= \pi r l$

$$l = \sqrt{12^2 + (3.5)^2} = 12.5 \text{ m} \qquad (1)$$

$\therefore$ Area of canvas required $= \frac{22}{7} \times 12 \times 12.5$

$$= 471.4 \text{ m}^2 \qquad (1)$$

14. Volume of sphere = volume of cone (1)

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{1}{3} \times \pi \times 6 \times 6 \times 24 \qquad (1)$$
$$\Rightarrow \quad r^3 = 6 \times 6 \times 6 \quad \Rightarrow \quad r = 6$$

$\therefore$ Radius of sphere = 6 cm (1)

15. Volume of coin $= \frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10}\ \text{cm}^3$ (1)

$|\because$ A coin is like a cylinder

Let the number of coins need to melt be 'n'

$$\therefore\ n = \left[10 \times \frac{55}{10} \times \frac{35}{10}\right] \div \left[\frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10}\right] \quad (1)$$

$$= 10 \times \frac{55}{10} \times \frac{35}{10} \times \frac{7}{22} \times \frac{200}{175} \times \frac{200}{175} \times \frac{10}{2} = 400$$

Thus, the required number of coins = 400. (1)

16. $\frac{3}{4} \times$ Volume of conical vessel

= Volume of cylindrical vessel (1)

Let the height of cylindrical vessel be h

$$\Rightarrow \frac{3}{\cancel{4}} \times \frac{1}{3} \times \pi \times 5 \times 5 \times \overset{6}{\cancel{24}} = \pi \times 10 \times 10 \times h \quad (1)$$

$$\Rightarrow \quad h = \frac{3}{2}\text{cm or } 1.5\text{ cm} \quad (1)$$

17. Let the area that can be irrigated in 30 minute be A m^2

Water flowing in canal in 30 minutes

$$= \left(10000 \times \frac{1}{2}\right)\text{ m} = 5000\text{ m}$$

Volume of water flowing out in 30 minutes

$$= (5000 \times 6 \times 1.5)\ \text{m}^3$$

$$= 45000\ \text{m}^3 \quad \ldots(i)\ (1)$$

Volume of water required to irrigate the field

$$= \text{A} \times \frac{8}{100}\ \text{m}^3 \quad \ldots(ii)\ (1)$$

Equating (i) and (ii), we get

$$\text{A} \times \frac{8}{100} = 45000$$

$$\Rightarrow \quad \text{A} = 562500\ \text{m}^2. \quad (1)$$

18. Volume of water in cylindrical tank

= Volume of water in park (1)

$$\Rightarrow \frac{22}{7} \times 1 \times 1 \times 5 = 25 \times 20 \times h \quad (1)$$

where h is the height of standing water

$$\Rightarrow \quad h = \frac{11}{350}\text{m or } \frac{22}{7}\text{cm} \quad (1)$$

19. Volume of rain water on the roof

= Volume of cylindrical tank (1)

i.e. $\quad 22 \times 20 \times h = \frac{22}{7} \times 1 \times 1 \times 3.5 \quad (1)$

$$\Rightarrow \quad h = \frac{1}{40}\text{ m} = 2.5\text{ cm} \quad (1)$$

20. $r_1 = 6$ cm

$r_2 = 8$ cm

$r_3 = 10$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$ (1)

Volume of the resulting sphere

= Sum of the volumes of the smaller spheres

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 \quad (1)$$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\,(r_1^3 + r_2^3 + r_3^3)$$

$$\Rightarrow \quad r^3 = 6^3 + 8^3 + 10^3$$

$$\Rightarrow \quad r^3 = 1728$$

$$\Rightarrow \quad r = \sqrt[3]{1728}$$

$$\Rightarrow \quad r = 12\text{ cm}$$

Therefore, the radius of the resulting sphere is 12 cm. (1)

21. 1.69 m (5)

22. Diameter of the pipe = 20 cm

$$\Rightarrow \text{ Radius of the pipe } (r) = \frac{20}{2} = 10\text{ cm} \quad (1)$$

Since, the water flows through the pipe at 3 km/hr.

$\therefore$ Length of water column per hour (h)

= 3 km = 3 × 1000 m = 3000 × 100 cm (1)

= 300000 cm.

$\therefore$ Volume of water = $\pi r^2 h$

$$= \pi \times 10^2 \times 300000\ \text{cm}^3$$

$$= \pi \times 30000000\ \text{cm}^3 \quad (1)$$

Now, for the cylindrical tank,

Diameter = 10 m

$$\Rightarrow \text{ Radius (R)} = \frac{10}{2} = 5 \times 100\text{ cm} = 500\text{ cm}$$

Height (H) = 2 m = 2 × 100 cm = 200 cm

$\therefore$ Volume of the cylindrical tank

$= \pi R^2 H = \pi \times (500)^2 \times 200\ \text{cm}^3$ (1)

Now, time required to fill the tank

$$= \frac{[\text{Volume of the tank}]}{[\text{Volume of water flown in 1 hour}]}$$

$$= \frac{\pi \times 500 \times 500 \times 200}{\pi \times 30000000}\text{ hrs}$$

$$= \frac{5 \times 5 \times 2}{30}\text{ hrs} = \frac{5}{3}\text{ hrs} = \frac{5}{3} \times 60\text{ minutes}$$

= 100 minutes. (1)

23. The volume of water in the overhead tank equals the volume of the water removed from the sump.

Now, the volume of water in the overhead tank (cylinder)

$= \pi r^2 h = 3.14 \times 0.6 \times 0.6 \times 0.95\ \text{m}^3$ (1)

The volume of water in the sump when full

$= l \times b \times h = 1.57 \times 1.44 \times 0.95\ \text{m}^3$ (1)

The volume of water left in sump after filing tank

$= [(1.57 \times 1.44 \times 0.95)$

$\quad - (3.14 \times 0.6 \times 0.6 \times 0.95)]\ \text{m}^3$

$= (1.57 \times 0.6 \times 0.6 \times 0.95 \times 2)\ \text{m}^2$ (1)

So, the height of water left in the sump

$$= \frac{\text{Volume of water left in the sump}}{l \times b}$$

$$= \frac{1.57 \times 0.6 \times 0.6 \times 0.95 \times 2}{1.57 \times 1.44} = 0.475 \text{ m}$$

$= 47.5$ cm (1)

Also, $\frac{\text{Capacity of tank}}{\text{Capacity of sump}}$

$$= \frac{3.14 \times 0.6 \times 0.6 \times 0.95}{1.57 \times 1.44 \times 0.95} = \frac{1}{2}$$

Therefore, the capacity of the tank is half the capacity of the sump. (1)

24. Let radius of spherical ball, $r_1 = 0.5$ cm

Then volume of one spherical ball,

$$V_1 = \frac{4}{3} \times \pi r_1^3$$

$$\Rightarrow \quad V_1 = \frac{4}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 0.5 \quad (1)$$

$$\Rightarrow \quad V_1 = \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10}$$

$$\Rightarrow \quad V_1 = \frac{11}{21} \text{ cm}^3$$

So, volume of 9000 spherical balls

$$= 9000 \times \frac{11}{21} = \frac{99000}{21} \text{ cm}^3 \quad (1)$$

Radius of cylindrical vessel = 10 cm

Let height of water raised = h

$\because$ Volume of 9000 balls

= Volume of water raised

$$\Rightarrow \frac{99000}{21} = \pi r^2 h \quad (1)$$

$$\Rightarrow \frac{99000}{21} = \frac{22}{7} \times 10 \times 10 \times h \quad (1)$$

$$\Rightarrow \quad h = \frac{99000 \times 7}{21 \times 22 \times 10 \times 10}$$

$$\Rightarrow \quad h = 15 \text{ cm} \quad (1)$$

25. For pipe, $r = 1$ cm (½)

Length of water flowing in 1 sec,

$h = 0.7$ m $= 70$ cm (½)

Cylindrical Tank, R = 40 cm, rise in water level = H (½)

Volume of water flowing in 1 sec

$= \pi r^2 h = \pi \times 1 \times 1 \times 70 = 70\pi$ (½)

Volume of water flowing in 60 sec

$= 70\pi \times 60$ (1)

Volume of water flowing in 30 minutes

$= 70\pi \times 60 \times 30$ (½)

Volume of water in Tank

$= \pi r^2 H = \pi \times 40 \times 40 \times H$ (½)

Volume of water in Tank

= Volume of water flowing in 30 minutes

$\pi \times 40 \times 40 \times H = 70\pi \times 60 \times 30$ (½)

$H = 78.75$ cm (½)

Case Study Based Questions

I. 1. (*d*) 404.25 cm^3 **2.** (*b*) 89.83 cm^3

3. (*d*) Cylindrical glass, 314.42 cm^3

4. (*c*) 24 m **5.** (*b*) 38.5 m^2

Exercise 6.2

I. Very Short Answer Type Questions [1 Mark]

1. Multiple Choice Questions (MCQs)

Choose the correct answer from the given options:

(1) A cylindrical pencil sharpened at one edge is the combination of
(*a*) a cone and a cylinder (*b*) frustum of a cone and a cylinder
(*c*) a hemisphere and a cylinder (*d*) two cylinders

(2) A *surahi* is the combination of
(*a*) a sphere and a cylinder (*b*) a hemisphere and a cylinder
(*c*) two hemispheres (*d*) a cylinder and a cone

(3) In a right circular cone, the cross-section made by a plane parallel to the base is a
(*a*) Triangle (*b*) Circle (*c*) Square (*d*) None of these

(4) If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is
(*a*) $4\pi r^2$ (*b*) $3\pi r^2$ (*c*) $2\pi r^2$ (*d*) πr^2

2. Assertion-Reason Type Questions

In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(*b*) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(*c*) Assertion (A) is true but reason (R) is false.
(*d*) Assertion (A) is false but reason (R) is true.

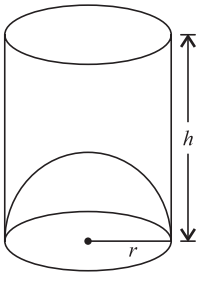
(1) **Assertion (A):** The number of coins 1.75 cm in diameter and 2 mm thick is formed from a melted cuboid 10 cm × 5.5 cm × 3.5 cm is 400.
Reason (R): Volume of a cylinder = $\pi r^2 h$ cubic units
and volume of cuboid = $(l \times b \times h)$ cubic units.

(2) **Assertion (A):** Number of spherical balls that can be made out of a solid cube of 1ead whose edge is 44 cm, each ball being 4 cm in diameter is 2541.
Reason (R): Number of balls = $\frac{\text{Volume of one ball}}{\text{Volume of lead}}$.

3. Answer the following.

(1) What is the capacity of cylindrical vessel with the hemispherical bottom portion raised upwards?

(2) A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of two parts are equal, then what is the ratio of its radius and the slant height of the conical part?

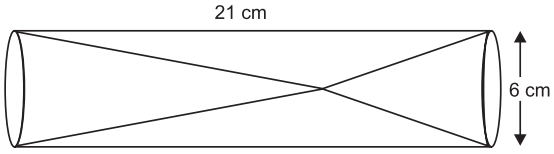


II. Short Answer Type Questions - I [2 Marks]

4. A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm. Find the total surface area of the toy. [NCERT][CBSE 2012]

5. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 58 cm and the diameter of the cylinder is 28 cm. Find the total surface area of the solid. [Use $\pi = \frac{22}{7}$] [CBSE 2006]

6. Two solid cones A and B are placed in a cylindrical tube as shown in the figure. The ratio of their capacities is 2 : 1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder. [NCERT Exemplar]

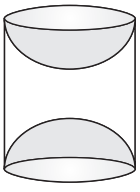


III. Short Answer Type Questions - II [3 Marks]

7. A toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the surface area of the toy. [Take $\pi = 3.14$]

8. From a solid cylinder of height 12 cm and diameter of the base 10 cm a conical cavity of the same height and same diameter is hollowed out. Find the surface area of the remaining solid. **[Imp.]**

9. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown: If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article. [NCERT]



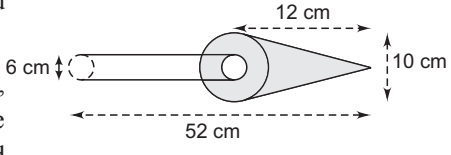
10. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and the radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if height of the conical part is 12 cm. **[Imp.]**

11. A *gulab jamun* when completely ready for eating contains sugar syrup upto about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns* shaped like a cylinder with two hemispherical ends, if the complete length of each of the *gulab jamuns* is 5 cm and its diameter is 2.8 cm. [NCERT][CBSE 2008]

12. A juice seller was serving his customers using glasses. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was 10 cm, find its actual capacity and its apparent capacity. (Use $\pi = 3.14$) [NCERT][CBSE 2009] **[Imp.]**

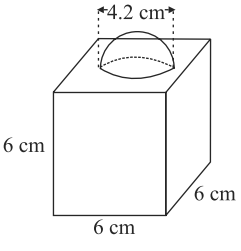
IV. Long Answer Type Questions [5 Marks]

13. The following figure shows a model which is the combination of a cone and cylinder.



The total length of the model is 52 cm, the length of conical portion is 12 cm, the base of the conical portion has diameter of 10 cm and the base of the cylindrical portion is of diameter 6 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the model painted with each of these colours.

14. In the given figure, a decorative block is shown which is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 6 cm and the hemisphere fixed on the top has a diameter of 4.2 cm. Find



(*a*) the total surface area of the block.

(*b*) the volume of the block formed [Take $\pi = \frac{22}{7}$] [CBSE 2019 (AI)]

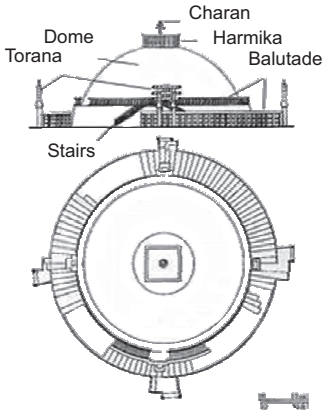
15. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of cylinder. The diameter and height of cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of rocket. (Use $\pi = 3.14$) [NCERT Exemplar]

16. A solid right circular cone of height 120 cm and radius 60 cm is placed in a right circular cylinder full of water of height 180 cm such that it touches the bottom. Find the volume of water left in cylinder, if the radius of the cylinder is equal to the radius to the cone. [NCERT Exemplar]

17. A solid is in the shape of a cone surmounted on a hemisphere. The radius of each of them being 3.5 cm and the total height of the solid is 9.5 cm. Find the volume of the solid.

Case Study Based Questions

I. The Great Stupa at Sanchi is one of the oldest stone structures in India, and an important monument of Indian Architecture. It was originally commissioned by the emperor Ashoka in the 3rd century BCE. Its nucleus is a simple hemispherical brick structure built over the relics of the Buddha. It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it. (Take = $\frac{22}{7}$)



1. The volume of the hemispherical dome if the height of the dome is 21 m, is

(*a*) 19404 cu. m (*b*) 2000 cu. m (*c*) 15000 cu. m (*d*) 19000 cu. m

2. The formula to find the volume of sphere is

(*a*) $\frac{2}{3}\pi r^3$ (*b*) $\frac{4}{3}\pi r^3$ (*c*) $4\pi r^2$ (*d*) $2\pi r^2$

3. The cloth required to cover the hemispherical dome if the radius of its base is 14 m is

(*a*) 1222 sq. m (*b*) 1232 sq. m (*c*) 1200 sq. m (*d*) 1400 sq. m

4. The total surface area of the combined figure, *i.e.* hemispherical dome with radius 14 m and cuboidal shaped top with dimensions 8 m × 6 m × 4 m is

(*a*) 1200 sq. m (*b*) 1232 sq. m (*c*) 1392 sq.m (*d*) 1932 sq. m

5. The volume of the cuboidal shaped top with dimensions mentioned in question 4, is

(*a*) 182.45 m^3 (*b*) 282.45 m^3 (*c*) 292 m^3 (*d*) 192 m^3

Answers and Hints

1. (1) (*a*) a cone and a cylinder (1)

(2) (*a*) a sphere and a cylinder (1)

(3) (*b*) Circle (1) (4) (*a*) $4\pi r^2$ (1)

2. (1) (*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

$$\text{Number of coins} = \frac{\text{Volume of cuboid}}{\text{Volume of one coin}}$$

$$= \frac{10\times5.5\times3.5}{\pi\times\frac{1.75}{2}\times\frac{1.75}{2}\times0.2}$$

$$= \frac{10\times5.5\times3.5}{\frac{22}{7}\times\frac{1.75}{2}\times\frac{1.75}{2}\times0.2}$$

$$= 400 \quad (1)$$

(2) (*c*) Assertion (A) is true but reason (R) is false.

We have,

$$\text{Number of balls} = \frac{\text{Volume of lead}}{\text{Volume of one ball}}$$

$$= \frac{44\times44\times44}{\frac{4}{3}\times\frac{22}{7}\times2\times2\times2}$$

$$= \frac{44\times44\times44\times3\times7}{4\times22\times2\times2\times2}$$

$$= 2541 \quad (1)$$

3. (1) $\frac{\pi r^2}{3}[3h-2r]$ (1) (2) 1 : 2 (1)

4. Here, $r = 3.5$ cm

$\therefore$ $h = (15.5 - 3.5)$ cm

$= 12.0$ cm

Surface area of the conical part = πrl

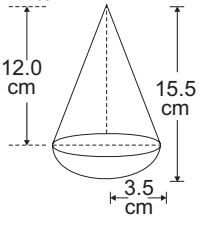
Surface area of the hemispherical part = $2\pi r^2$

$\therefore$ Total surface area of the toy

$= \pi rl + 2\pi r^2 = \pi r(l+2r)$ cm^2

$\because$ $l^2 = (12)^2 + (3.5)^2 = 156.25$ cm^2

$\Rightarrow$ $l = 12.5$ cm (1)



∴ TSA of the toy

$= \frac{22}{7} \times \frac{35}{10} (12.5 + 2 \times 3.5)\ \text{cm}^2$

$= 11 \times (12.5 + 7)\ \text{cm}^2$

$= 11 \times 19.5\ \text{cm}^2$

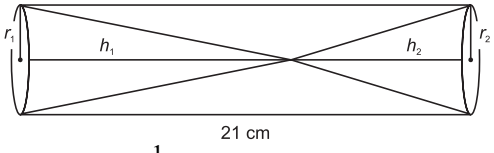
$= 214.5\ \text{cm}^2$. (1)

5. Total surface area of solid = Curved surface area of cylindrical portion + 2(Curved surface area of hemispherical portion)

$= 2 \times \frac{22}{7} \times 14 \times 30 + 2 (2 \times \frac{22}{7} \times 14 \times 14)$ (1)

$= 2640 + 2464 = 5104\ \text{cm}^2$ (1)

6. As the ratio of volumes of cones A and B is 2 : 1, their radii are same equal to $r = \frac{6}{2} = 3$ cm.



$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} \Rightarrow \frac{2}{1} = \frac{(3)^2 h_1}{(3)^2 h_2}$

$\Rightarrow \quad h_1 = 2h_2$ ...(*i*)

Also, $h_1 + h_2 = 21$ cm

$\Rightarrow \quad 2h_2 + h_2 = 21$

$\Rightarrow \quad 3h_2 = 21$ [Using (*i*)]

$\Rightarrow \quad h_2 = 7$ cm

Now, $h_1 = 21$ cm $- 7$ cm $= 14$ cm ...(*ii*)

Hence, height of cone A = 14 cm and height of cone B = 7 cm. (1)

Volume of cone A $= \frac{1}{3} \times \frac{22}{7} \times 9 \times 14 = 132\ \text{cm}^3$

Volume of cone B $= \frac{1}{3} \times \frac{22}{7} \times 9 \times 7 = 66\ \text{cm}^3$

Volume of remaining portion of tube

= Vol. of cylinder – Vol. of cone A – Vol. of cone B

$= \pi r^2 h - 132\ \text{cm}^3 - 66\ \text{cm}^3$

$= \frac{22}{7} \times 3 \times 3 \times 21 - 198$

$= 22 \times 27 - 198 = 594 - 198 = 396\ \text{cm}^3$

Hence, the required volume is 396 cm^3. (1)

7. Surface area of toy

$= 2\pi \times 3 \times 3 + \pi \times 3 \times \sqrt{3^2 + 4^2}$ (1½)

$= 18\pi + 15\pi = 33\pi = 33 \times \frac{22}{7}$

$= 103.62\ \text{cm}^2$ (1½)

8. Surface area of remaining solid

$= 2\pi \times 5 \times 12 + \pi \times 5 \times 5 + \pi \times 5 \times \sqrt{5^2 + 12^2}$ (1)

$= 120\pi + 25\pi + 65\pi$ (1)

$= 210\pi = 210 \times \frac{22}{7} = 660\ \text{cm}^2$ (1)

9. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

∴ Total surface area

$= 2\pi rh + 2\pi r^2 + 2\pi r^2$ (1)

$= 2\pi r (h + r + r) = 2\pi r (h + 2r)$

$= 2 \times \frac{22}{7} \times 3.5 (10 + 2 \times 3.5)$ (1)

$= 2 \times \frac{22}{7} \times 3.5 \times 17 = 374\ \text{cm}^2$ (1)

10. 770 cm^2 (3)

11. 337.88 cm^3 (3)

Hint: Radius (r) $= \frac{2.8}{2}$ cm $= 1.4$ cm

⇒ length (h) of cylindrical part

$= 5\ \text{cm} - 2 \times 1.5\ \text{cm} = 2.2\ \text{cm}$

Volume of each *gulab jamun*

$= \pi(1.4)^2 \times 2.2 + 2 \times \frac{2}{3}\pi \times (1.4)^3\ \text{cm}^3$

Volume of syrup found in 45 *gulab jamuns*

= 45 × 30% of volume of each *gulab jamun*.

12. Since the inner diameter of the glass = 5 cm and height = 10 cm, the apparent capacity of the glass = $\pi r^2 h$ $= 3.14 \times 2.5 \times 2.5 \times 10\ \text{cm}^3 = 196.25\ \text{cm}^3$ (1)

But the actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.

i.e., it is less by $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5\,\text{cm}^3 = 32.71\ \text{cm}^3$ (1)

So, the actual capacity of the glass

= apparent capacity of glass – volume of the hemisphere

$= (196.25 - 32.71) = 163.54\ \text{cm}^3$ (1)

13. orange: 254.34 cm^2, yellow: 781.86 cm^2 (5)

Hint: For conical portion, $h = 12$ cm,

$r = \frac{10}{2}$ cm = 5 cm

and slant height (l) = $\sqrt{h^2 + r^2}$ = 13 cm;

whereas for cylindrical portion,

$h = (52 - 12)$ cm = 40 cm and $r = \frac{6}{2}$ cm = 3 cm.

$\therefore$ Area to be painted orange

= C.S.A. of the cone + Area of its base

$= \pi \times 5 \times 13 + \pi (5^2 - 3^2)$ sq. cm

Area to be painted yellow

= C.S.A. of the cylinder + Area of its base

$= 2\pi \times 3 \times 40 + \pi 3^2$ sq. cm

14. Diameter of the hemisphere = 4.2 cm

$\therefore$ Radius of the hemisphere = 2.1 cm

Edge of the cube (l) = 6 cm (1)

(a) Total surface area of the block

= Total surface area of the cube

– Base area of hemisphere

+ Curved surface area of hemisphere

$= 6 \times (\text{edge})^2 - \pi r^2 + 2\pi r^2$

$= 6 l^2 + \pi r^2 = 6 \times 36 + \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10}$

$= 216 + 13.83 = 229.86\ cm^2$ (2)

(b) Volume of the block

= Volume of cube + Volume of hemisphere

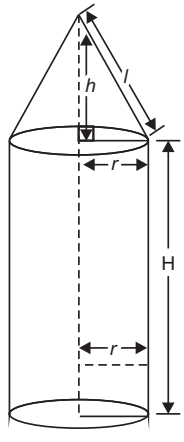
$= (\text{edge})^3 + \frac{2}{3}\pi r^3 = (6)^3 + \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{10}\right)^3$

$= 216 + \frac{2}{3} \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times \frac{21}{10}$

$= 216 + 19.40 = 235.40\ cm^3$ (2)

15. Volume of rocket

= Volume of cylinder + Volume of cone



(1)

$= \pi r^2 H + \frac{1}{3}\pi r^2 h = \pi r^2 \left[H + \frac{1}{3}h\right]$

$= 3.14 \times 3 \times 3 \left[12 + \frac{1}{3} \times 4\right]$ (1)

[$\because h = \sqrt{5^2 - 3^2} = 4$ cm]

$= 3.14 \times 9 \left[\frac{40}{3}\right] = 3.14 \times 3 \times 40 = 376.8\ cm^3$

$\therefore$ Volume of Rocket = 376.8 cm^3 (1)

Total surface area of rocket

= Curved surface area of cylinder

+ Curved surface area of cone

+ Area of base of cylinder

[As it is closed (Given)]

$= 2\pi rH + \pi rl + \pi r^2 = \pi r\,[2H + l + r]$ (1)

$= 3.14 \times 3[2 \times 12 + 5 + 3] = 3.14 \times 3 \times 32$

$= 301.44\ cm^2$ (1)

Hence, the surface area of rocket is 301.44 cm^2.

16. The water left in cylinder

= Volume of cylinder – Volume of cone

Volume of water left after immersing the cone into cylinder full of water

= Volume of cylinder – Volume of cone (1)

$= \pi R^2 H - \frac{1}{3}\pi r^2 h$

$\therefore$ Required volume of water in cylinder

$= \pi r^2 H - \frac{1}{3}\pi r^2 h$ [$\because R = r$] (1)

$= \pi r^2 \left[H - \frac{1}{3}h\right] = \frac{22}{7} \times 60 \times 60 \left[180 - \frac{120}{3}\right]$

$= \frac{22}{7} \times 60 \times 60 \times 140\ cm^3$ (1)

$= \frac{22 \times 60 \times 60 \times 140}{7 \times 100 \times 100 \times 100} = \frac{22 \times 72}{1000} = \frac{1584}{1000}$

$\therefore$ Vol. of water in cylinder = 1.584 m^3

Hence, required volume of water left = 1.584 m^3. (1)

17. Let the radius of hemisphere or a cone, r = 3.5 cm

Also, the height of the cone,

$h = 9.5 - 3.5 = 6$ cm (2)

Volume of solid

= Volume of hemisphere + volume of cone

$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$ (1)

$= \frac{2}{3} \times \frac{22}{7} \times (3.5)^3 + \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6$

$= 89.83 + 77 = 166.83\ cm^3$ (2)

Case Study Based Questions

I. 1. (a) 19404 cu. m **2.** (b) $\frac{4}{3}\pi r^3$

3. (b) 1232 sq. m **4.** (c) 1392 sq. m

5. (d) 192 m^3